Anomalous pulse delay in microwave propagation: A stochastic process interpretation

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An experiment involving microwave propagation in the near-field region with two horn antennas demonstrated a superluminal behavior which is strongly dependent on the frequency. The models previously proposed are found to be inadequate for interpreting the results. An attempt is made within the framework of a stochastic model, which can be improved by a path-integral analysis.

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Delay time measurements of microwave pulses, using horn antennas in open air propagation, have been extended to different frequencies still in the near-field region. The average delay decreases when the receiver is shifted with respect to the launcher, enhancing superluminal behavior, that is, motion faster than the speed of light in vacuum. However, this effect is strongly frequency dependent. The delay shows a typical undulating shape, the interpretation of which, on the basis of the model previously proposed for explaining this phenomenon at a fixed frequency [1], is arduous. The interpretation proposed here is mainly based on a modified stochastic (path-integral) model that has already demonstrated an ability to interpret tunneling processes [2,3], that is, situations in which evanescent waves are dominant. Special kinds of decaying waves are the (proper) complex waves [4], the field of which can be expressed in polar coordinates as $\sim 2\pi i \exp[ik\rho\cos(\beta+\alpha)]$, where $k=2\pi/\lambda$, ρ is the distance, α is the angle of observation, and β is a complex angle whose real part β_r is comparable to one-half of the flare angle of the launcher (see Fig. 1). These waves can be sufficiently persistent, and may prevail over the normal field, up to distances of tens of wavelengths, that is, up to distances of the order of 1 m, for microwaves in the X band ($\lambda \approx 3$ cm) and even more for longer wavelengths [5]. In Fig. 1 we note that the complex waves that we are considering actually present both the characteristics of normal waves and those of decaying waves, especially when the sum of angles $\alpha+\beta_r$ is sufficiently large. For $\alpha+\beta_r \to \pi/2$, they behave exactly like evanescent waves [1].

The shortening of the pulse delay, that is observed when the receiver horn is shifted with respect to the launcher was interpreted on the basis of a complex-wave analysis which led to a cos $(\alpha+\beta)$ dependence. Other interpretations have been proposed, starting from the work by Giakos and Ishii [6] who first observed this effect. Another interpretation, which is given in Ref. [7], is based on the helicity at the photon motion being due to some "exotic" force, and invokes as proof the existence of an angular momentum associated with the radiation. As is well known, the latter has been known since the pioneering work by Beth [8] and Carrara [9], and recently by He *et al.* [10]. However, we doubt that this argument can be adopted for interpreting these kinds of experiments. Nevertheless, the concept of a frequency threshold, which is inherent in the model of Ref. [7], and which was reconsidered very recently by assimilating the system to a high-pass filter [11], seems to have a role also in the interpretation that we are proposing here in the light of our results.

The experimental setup is essentially the same as the one adopted in Refs. [1] and [5]. The experiment consisted of an "open-air" pulse transmission between two horn antennas (mouth sizes 20×15 cm^2; flare angle 54°) separated by a distance $L=60$ cm. A microwave signal like a step function was supplied by a generator modulated by a pin modulator, the fall time of which (~ 10 ns) was suitable for measuring delay times down to less than 1 ns. The signals taken at the launcher and at the receiver (the spatial separation was $D=110$ cm) were sent to a high-temporal-resolution oscilloscope that was able to measure the delay with an accuracy of ± 0.1 ns. The results obtained in the range of frequency between 4 and 7 GHz are shown in Fig. 2. Initially, the launcher and the receiver horns were facing each other. In this condition, we measured an average pulse delay of about

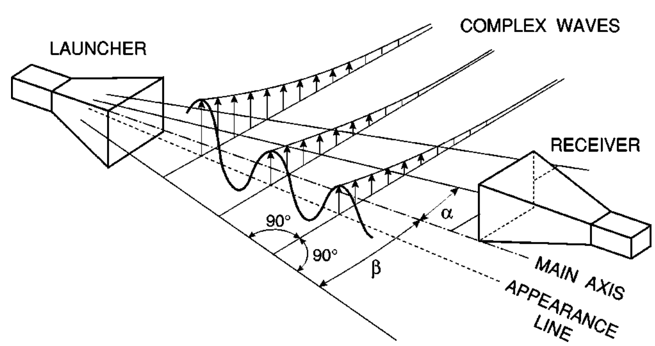

FIG. 1. In the propagation experiment with horn antennas, a special kind of wave, the complex wave, is operating in the near-field limit as can be detected by a receiver in a selected range of values of the angle α. The wave fronts are perpendicular to a plane nearly coincident with the vertical wall of the launcher forming the angle β with the axis of the horn. When the receiver is displaced, the sum of the angles $\alpha+\beta$ increases and the observed delay decreases according to a cos$(\alpha+\beta)$ dependence, after Ref. [5].

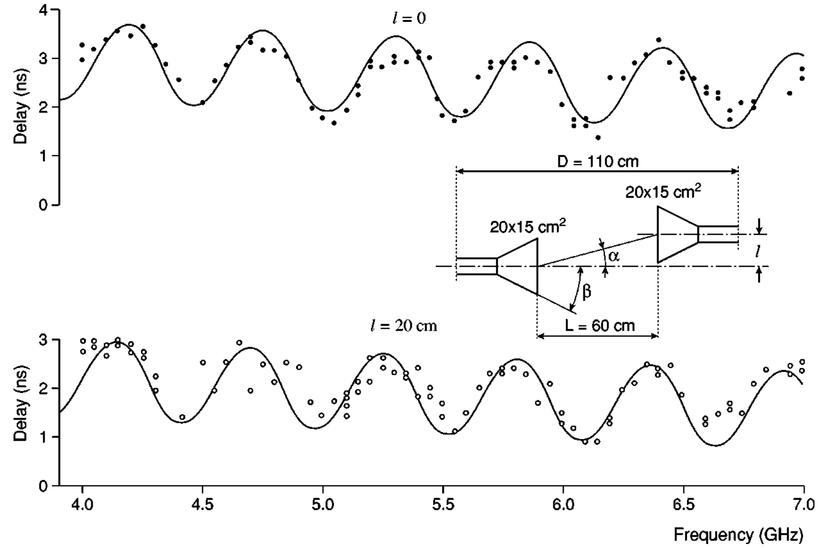

FIG. 2. Pulse delay in propagation experiments with two horn antennas as a function of the frequency in the 4–7 GHz range, measured at fixed values of the displacement l. For $l=0$, in the upper portion of the figure, we measured (full circles) an average delay of about 2.5 ns, 2 ns of which would correspond to traveling the distance $L=60$ cm between the two horns at the velocity of light in vacuum. For $l=20$ cm, in the lower portion of the figure, the measured delay (open circles) is slightly shortened by about 0.5 ns. In both cases, we have an undulating behavior, the period of which is nearly constant with the frequency, and an almost constant amplitude which, by taking into account an offset zero of $\sim$1.5 ns, varies from $\sim$2 to $\sim$0.5 ns: that is, between a value that is almost coincident with the case of normal propagation and a situation of strong superluminal behavior. The reported curves, obtained from the expression given in the text, are obtained with $L/v \simeq 1.8$ ns and zero offset $A \simeq 1$–1.5 ns.

2.5 ns, 2 ns of which could be attributed to the delay involved in traveling the separation $L=60$ cm at light speed c. The receiver horn was then shifted transversally. Similarly to what has been observed in analogous experiments [1,5], on increasing the perpendicular displacement l, the measured delay tended to decrease. For $l=20$ cm, the average delay was about 2 ns, i.e., shortened by $\sim$0.5 ns, in rough agreement with the $\cos(\alpha+\beta)$ dependence mentioned above. In both cases, the delay as a function of the frequency (see Fig. 2) exhibited a marked undulating shape with a period that was nearly independent of the frequency. The almost constant amplitude ($\sim$1.5 ns) also showed a strong variation in the delay by taking into account a zero offset of about 1.5 ns, due to the travel in the two horns, as can be seen from the data obtained for $l=0$ (which should have corresponded to an average delay of $\sim$2 ns, but we found it to be measurably smaller). Therefore, we have the result that the delay varied from values that were comparable with the one for which the velocity is c and values close to zero, which exhibited a evident superluminal behavior. Roughly speaking, the average delay was about one-half the delay corresponding to the light velocity c.

These results are rather puzzling and certainly unexpected in light of the models considered up until now. At first sight, we noted that the periodicity of the delay corresponded to the variation in the number of wavelengths comprised within the distance L between the launcher and the receiver (for instance, at 5 GHz 10 wavelengths were comprised in 60 cm, 11 at 5.5 GHz, and so on). Given the spatial dependence of the waves involved, a first interpretation of the behavior observed can be attempted within the framework of reflection of normal waves, although limited to the region of their existence, which is that of the near field.

The periodicity in the measured delay suggests the existence of a series of resonances, analogously to what has been observed in resonant tunneling [12], but here the situation is different, since it is that of wave reflection. We follow the analysis of Ref. [13] relative to a finite, asymmetric, square potential well which predicts the existence of a series of reflection resonances, (see Fig. III.5 in Ref. [13]), to which a series of peaks in the traversal time also corresponds. Really, in Ref. [13] the reflection time is considered; however, it can be shown that this time also corresponds to the traversal time [14]. By adapting these results to our case (see Fig. 3) (where the Schrödinger equation becomes the Helmholtz equation), and by putting $\eta^2=(\nu-\nu_0)/\nu_0$, $K_0=2\pi/\lambda_0$ where ν_0 is a cutoff frequency and λ_0 the corresponding wavelength, we have that the separation between the peaks is given by $\Delta\eta^2=2\pi/K_0L$, where L is the width of the well (in our case the distance between the two horns), provided that $K_0L \gg \pi$ and $\eta \ll 1$. Taking $\nu_0=3$ GHz (this value is justified later on) and $L=60$ cm ($K_0L \simeq 38$), we obtain $\Delta\eta^2=0.166$, which corresponds to $\Delta\nu$=0.5 GHz. This value is in excellent agreement with the observations in Fig. 2, where the period of the undulating delay is about 0.5 GHz. Another important result is the following. As previously anticipated, the period of the delay corresponds to variations of *one* wavelength. Since the resonance frequencies are given by $\omega_n=(n+1/2)\pi v/L$, where n is an integer number, so that $\nu_n=(n/2+1/4)v/L$, in order to obtain $\nu_n \simeq 4.25, 4.75, 5.25, \ldots,$ which are the approximate positions of the maxima in Fig. 2, for $L=60$ cm, we need a velocity

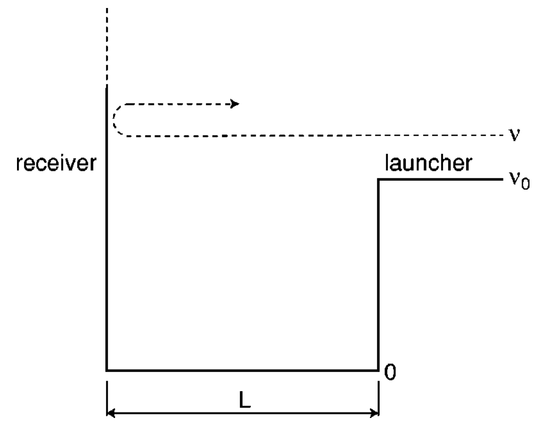

FIG. 3. Schematic representation of wave reflection in the space between the two horn antennas.

$v \simeq 2c$, a value which is contained between the extrema of the measured delay. Similar conclusions can be drawn also for the results reported in Fig. 4 of Ref. [5], where we can envisage an undulation in the measured delay vs frequency, in the 1.2–2 GHz range, with a period of about 0.1 GHz. There, taking $\nu_0 = 1$ GHz, $L = 338$ cm ($k_0 L = 70$), we obtain $\Delta\nu \approx 0.09$ GHz, which again is in good agreement with observation.

A more refined (in a sense complementary) interpretation can be envisaged by adapting the stochastic model already formulated for tunneling processes. This assumption is justified by the fact that, even if not properly evanescent as in tunneling cases, the waves involved are decaying, and survive only in the near-field region. In its simplified version, the stochastic approach supplies for the delay time a complex quantity, the real part (that directly observable) of which is given by [15]

$$\mathrm{Re}\langle t \rangle = \frac{1}{2a}\left[1 - \cos\left(2a\frac{L}{v}\right)\right], \tag{1}$$

where a is the dissipative parameter entering the telegrapher equation, which is the basis for this kind of treatment [2], and L can be taken to be coincident with the separation between the two horns. Assuming that a is less than, but comparable with, ω and that the velocity v is greater than $c = \lambda\nu$, the argument of the cosine in Eq. (1) can be put in a form such as $2\pi L/\lambda = 2\pi\nu L/v$, where L/v must be considered as an adjustable parameter.

The curves reported in Fig. 2 were obtained at first by calculating the expression

$$A[1 - \cos(2\pi\nu L/v)] + B, \tag{2}$$

where $A \approx 0.7$ is an amplitude factor, $B \simeq 1$–1.5 ns is due to the zero offset mentioned above, and $L/v = 1.8$ ns. This value implies $a \simeq \omega/2 = 14$–22 (ns)$^{-1}$, values that are comparable with the ones adopted in tunneling cases [2,3]. The curves obtained give a rough but acceptable description of the experimental data. The only serious discrepancy with the model comes from disregarding the factor $1/2a$ in Eq. (1). This could be partially justified by considering that the observed effect of shortening in the delay may be a little overestimated (see footnote [7] in Ref. [5]). However, the most important cause of discrepancy is the adoption of the model of Eq. (1) which is surely oversimplified for our case and can be considered only as indicative. Therefore, a further attempt at perfecting the data analysis and the model would seem to be worthwhile.

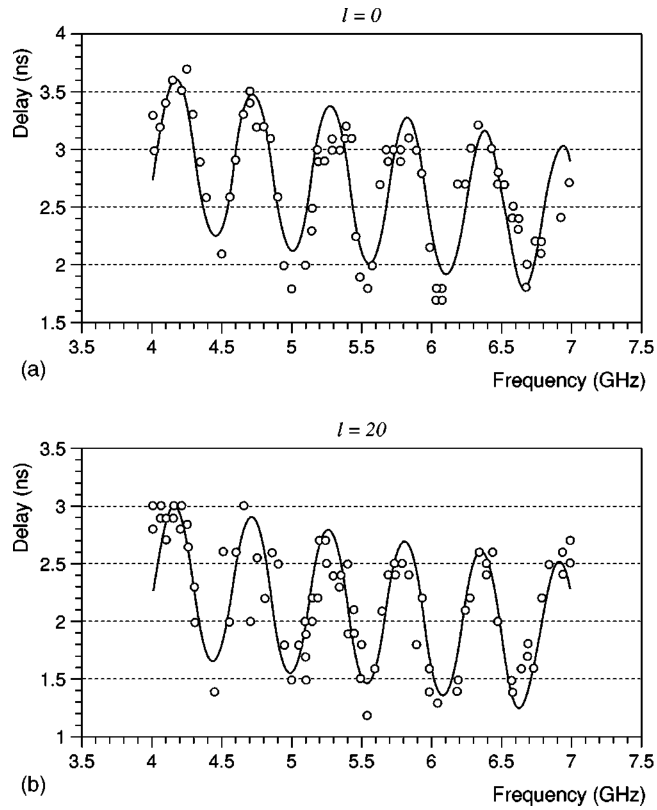

FIG. 4. The same data as in Fig. 2 superimposed on the curves (2), as determined by a best-fitting procedure, the parameter values of which are given in Table I.

The results of a more precise data analysis are shown in Fig. 4, where the curves were obtained with a best fitting of expression (2) and where we have adopted the form $B = C - D(\nu - \nu_i)$, with $\nu_i = 4$ GHz, for the zero offset. The best fits of the data were obtained with the parameter values reported in Table I. These values are comparable with the ones of the preliminary analysis. The values obtained for the correlation coefficient, $R \simeq 0.84$ for the upper curve ($l = 0$) in Fig. 4 and $R \simeq 0.80$ for the lower curve ($l = 20$ cm), represent a plausible data description obtained by the adopted model.

TABLE I. Parameter values and error analysis relative to the curves and the data of Fig. 4 as given by a best-fitting procedure, where $B = C - D(\nu - \nu_i)$, $\nu_i = 4$ GHz.

l(cm)	A(ns)	L/v(ns)	C(ns)	D(ns/GHz)	χ^2	R
0	0.65	1.7975	2.3405	0.2055	5.7156	0.8405
20	0.65	1.8059	1.7280	0.1772	6.9529	0.7988

Now we shall consider perfecting the theoretical model. An attempt in this direction can be made within the framework of the path-integral method and, in particular, by using a transition-element analysis [16]. Here, we limit ourselves to giving some estimates and to providing the preliminary results of a rather complicated procedure. A full account will be given elsewhere.

Let us assume that the effect of the dissipation can be accounted for by writing the action in the form

$$S' = S + \int f(t)x(t)dt, \tag{3}$$

where S is the unperturbed action, $f(t)$ is any arbitrary function of the time, and $x(t)$ is a trajectory. This leads to a result, as expressed by Eq. (7.69) in Ref. [16], in which $x(t)$ is shown to be modulated by a factor whose real part is given by $\cos[(S'-S)/\hbar]$.

According to a phenomenological approach to the dissipative effects [17], the energy losses are given by $W(x) = \eta\int_0^x|\dot{x}|dx$, where $\eta = 2am$ is the dissipative constant and m is the mass of the particle, which, under certain acceptable assumptions, can be approximated as $W \approx \eta|\dot{x}|x$. The variation of the action is then given by

$$\Delta S = S' - S = \eta\int |\dot{x}(t)|x(t)dt, \tag{4}$$

where in our case, by comparison with Eq. (3), we can identify $f(t)$ with $|\dot{x}(t)|$. Under the same assumption as before, we have $\Delta S \approx \eta\omega|x|^2 t$, as $|\dot{x}| = \omega|x|$.

This rough estimate is confirmed by the result expressed by Eq. (5) in Ref. [18] for a damped harmonic oscillator (which includes our case), which reads as

$$\Delta S = \frac{amx_0^2}{2}[\omega t\sin(2\omega t) + \cos(2\omega t)], \tag{5}$$

x_0 being the amplitude of the pseudo oscillations ($2x_0 = L$ in our case). Here, by disregarding the rapid oscillation due to the angular frequency ω, that is, by considering the absolute value in Eq. (5), we obtain the approximate result $|\Delta S| \approx \frac{1}{2}amx_0^2\omega t$, which is comparable to the previous estimate. These results, which are relative to a mechanical oscillator, can be translated into the electromagnetic framework by using the correspondence $mc^2/\hbar \rightarrow \omega$ [19]. However, we can follow a simplified approach by considering that, for the unperturbed action $S = \frac{1}{2}m\omega x_0^2$, we take the value $N\hbar$. Here, N is (presumably) a relatively large number, so we ultimately obtain $|\Delta S|/\hbar \approx Nat$. By identifying, as before, the time t with L/v, we recover an expression that is very similar—apart from a suitable zero offset– to Eq. (1). More precisely, in accordance with a functional analysis, it can be shown [20] that the real part of the transition element of the time, within the limit of high values of ω, is given by

$$\mathrm{Re}\langle t\rangle \simeq \frac{L}{v}\left[1 - \frac{a}{2\omega}\left(\frac{v}{c}\right)^2\cos\left(2a\frac{L}{v}\right)\right]\langle 1\rangle, \tag{6}$$

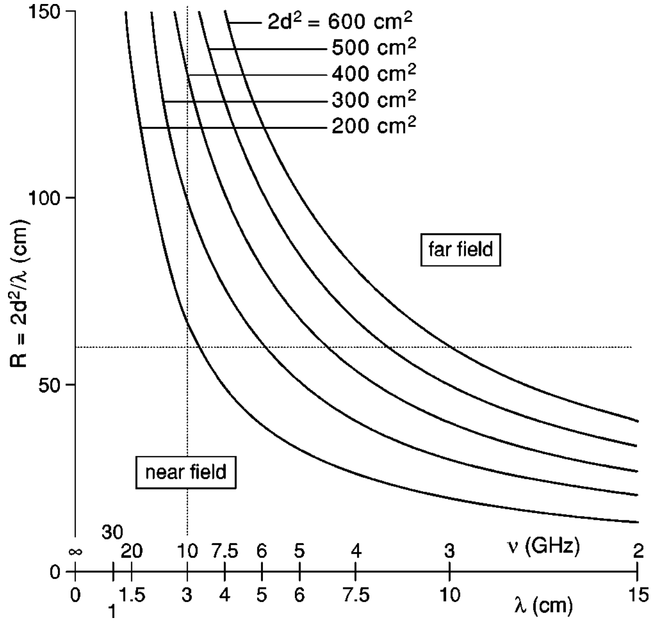

FIG. 5. Borderlines of the near-field region, calculated for some values of $2d^2$ as a function of the wavelength or of the frequency. For $R = L = 60$ cm (horizontal dotted line) and $2d^2 \simeq 600$ cm^2, the frequency which delimits the near field is ~ 3 GHz. For a fixed value of the frequency, e.g., $\nu = 10$ GHz (vertical dotted line) and $2d^2 \simeq 200$ cm^2, as in Refs. [1] and [5], the range which delimits the near field is $R \simeq 70$ cm.

where $\langle 1\rangle$ signifies a propagator that can be related to the attenuation of the waves according to the relation

$$\langle 1\rangle \propto \exp[-k\rho\sin(\beta_r + \alpha)\sinh\beta_i], \tag{7}$$

where $k = 2\pi/\lambda$, $\rho = \sqrt{L^2 + l^2}$, and β_r and β_i are (we recall) the real and imaginary parts, respectively, of the angle β [1,5]. In spite of this (moderate) effect, Eq. (6) actually resembles Eq. (1). However, the amplitude factor $(L/v)\langle 1\rangle$, as well as the quantity $(a/2\omega)(v/c)^2$ multiplying $\cos(2aL/v)$ in Eq. (6), are now of the right order of magnitude. In fact, by depending on a more accurate selection of the several quantities, the matching with the parameter values of the curves fitting the data can be obtained in a range of values. Thus, for $L/v = 1.8$ ns we need $a = \omega/2$, but by lowering L/v to ~ 1 ns, as required by the resonance condition discussed before, we should take $a \approx \omega$. For the propagator $\langle 1\rangle$, this requires values comprised in the 1/3–2/3 interval, values which are plausible as well. Analogously, the quantity $(a/2\omega)(v/c)^2$ varies in the 0.25–2 range. It is, therefore, merely a question of adjusting these values in order to obtain a perfect matching with the fitting parameters.

Finally, we wish to justify the cutoff frequency for these kinds of systems. As firmly established here and in previous experiments [1,5] the observed behavior is confined to the near-field region. According to a conventional definition [21], the borderline which roughly delimits the near field is given by $R = 2d^2/\lambda$, R being the distance and d the width of the antenna aperture, and λ the wavelength. In the range of values of interest to us, in Fig. 5 we report some curves calculated for certain values of $2d^2$. We note that, for $R=L=60$ cm and for $2d^2 \approx 600$ cm^2 (that is, twice the area of each mouth), the frequency which delimits the near field is $\sim$3 GHz, a value that is coincident with the adopted ν_0. With increasing frequency, we enter the near-field region, thus confirming the nature of the observed facts and, in a sense, the high-pass filter behavior hypothesized elsewhere [11].

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